A Phenomenological Analysis of Higher Fock State Contributions to the χ_{cJ} Decays*

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Abstract

We present a phenomenological analysis of higher Fack state contributions to the χ_{cJ} decays by using the recent BES experimental data. It is found that the higher Fock state $|\ (c\bar{c})_8 g >$ makes an important contributions to the inclusive and exclusive processes with respect to that from the valence Fock state $|\ c\bar{c} >$ of the χ_{cJ} and some constraints of these contributions are obtained for the χ_{c0} and χ_{c2} states in order to fit the experimental data.

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1. Introduction

Recently, BES has reported about 30 channels of χ_{cJ} hadronic decay[1] and the decays to $\pi\pi, KK$ among them have improved greatly with high precision[2]. For example, some branching ratios are shown in the following table.

Decay channels	BES ($\times 10^{-3}$)
$Br(\chi_{c0} \to \pi^+\pi^-)$	$4.68 \pm 0.26 \pm 0.65$
$Br(\chi_{c2} \to \pi^+\pi^-)$	$1.49 \pm 0.14 \pm 0.22$
$Br(\chi_{c0} \to K^+K^-)$	$5.68 \pm 0.35 \pm 0.85$
$Br(\chi_{c2} \to K^+K^-)$	$0.79 \pm 0.14 \pm 0.13$

χ_{cJ} are the P-wave bound state of $c\bar{c}$ and can be treated approximately as a non-relativistic system due to the large mass of the charm quark. In such a system, the charm quark and anti-charm quark are coupled to light quarks by two-gluon exchange which has large momentum flow. Then these light quarks will build up the products of the decay, i.e. light hadrons. The total hadronic widths of χ_{cJ} are approximately equal to the widths of the decays into two gluons[3],

$$\Gamma^{(1)}_{tot}(\chi_{c0}) = \frac{6\ \alpha_s^2(M_c^2)}{M_c^4} \mid {R'}_p(0) \mid^2 \tag{1}$$

$$\Gamma^{(1)}_{tot}(\chi_{c2}) = \frac{8\ \alpha_s^2(M_c^2)}{5M_c^4} \mid R_p^{'}(0) \mid^2 \tag{2}$$

where $R_p^{'}(0)$ is the derivative of the radial wavefunction of the heavy quankonium at origin. Here we only consider the valance Fock state contribution. Theorefore the ratio of the total decay widths

$$R \equiv \frac{\Gamma_{tot}(\chi_{c0})}{\Gamma_{tot}(\chi_{c2})} \cong \frac{\Gamma^{(1)}_{tot}(\chi_{c0})}{\Gamma^{(1)}_{tot}(\chi_{c2})} = \frac{15}{4} \tag{3}$$

after neglecting the v^2 corrections. It was assumed that the single non-perturbative quantity $R_p^{'}(0)$ includes the long-distance effects. However, Barbieri et al.[4] found that the coefficients of $| R_p'(0) |^2$ depend logarithmically on an infrared cutoff on the energies of the final-state gluons. Bodwin et. al.[5] pointed out that to calculate the coefficients of $| R_p'(0) |^2$ to relative order $\alpha_s^n(\alpha_s(Mv))$ should include all operators whose matrix elements are of relative order v^n or less. Thus the annihilation rate of the χ_{cJ} into light hadrons becomes[6]

$$\Gamma(\chi_{cJ}) = \sum_n \frac{2\ Im\ fn(\Lambda)}{M^{d_n - 4}} < \chi_{cJ} \mid O_n(\Lambda) \mid \chi_{cJ} > \tag{4}$$

and the dependence on the arbitrary factorization scale Λ in Eq.(4) cancels between the coefficients and the operators. The P-wave χ_{cJ} state can be expressed as

$$\mid \chi_{cJ} >= O(1) \mid (c\bar{c})_1(^3P_J) > +O(v) \mid (c\bar{c})_8(^3S_1)g > + \ O(v^2) \tag{5}$$

As keeping the first term in Eq.(5), Eq.(4) becomes Eqs.(1) and (2). They also show that the inclusion of the higher Fock states in the factorization formulas removes the dependence of the decay rate on an arbitrary infrared cutoff. Eq.(4) gives the corrections to the leading contribution Eqs.(1) and (2).

Now we consider the case of exclusive decays of χ_{cJ}, such as $\chi_{cJ} \to \pi\pi, KK$,... . Following the framework of calculations of exclusive processes at large momentum transfers[7], the decay amplitude of χ_{cJ} can be factorized into two parts: a hard amplitude T_H calculable in perturbative QCD, and the distribute amplitudes ϕ_H for each hadron H. For example, taking the valence Fock state in Eq.(5), the decay width of $\chi_{cJ} \to \pi\pi$ is given by

$$\Gamma^{(1)}(\chi_{c0} \to \pi\pi) = \frac{C^2[4\pi\ \alpha_s(M^2)]^4}{2\pi^2 \cdot 32M^8} \mid R_p'(0) \mid^2 \quad \mid I_0^\pi \mid^2 \tag{6}$$

and

$$\Gamma^{(1)}(\chi_{c2} \to \pi\pi) = \frac{C^2[4\pi\ \alpha_s(M^2)]^4}{5\pi^2 \cdot 32M^8} \mid R_p'(0) \mid^2 \quad \mid I_2^\pi \mid^2 \tag{7}$$

where

$$I_0^\pi = \int_0^1 dx \int_0^1 dy \phi_\pi(x, Q^2) \frac{2 + \frac{(x-y)^2}{x+y-2xy}}{x(1-x)(x+y-2xy)y(1-y)} \phi_\pi(y, Q^2) \tag{8}$$

$$I_2^\pi = \int_0^1 dx \int_0^1 dy \phi_\pi(x, Q^2) \frac{1 - \frac{(x-y)^2}{x+y-2xy}}{x(1-x)(x+y-2xy)y(1-y)} \phi_\pi(y, Q^2) \tag{9}$$

and

$$C = (\frac{1}{\sqrt{n_c}})^3 \sum_{AB} [Tr(T^A T^B)]^2 = \frac{2}{3\sqrt{3}} \tag{10}$$

is the color factor. The distribution amplitude $\phi_\pi(x, Q^2)$ in Eqs.(8) and (9) is defined by the valence wave function $\psi_{q\bar{q}/\pi}(x, k_\perp)$ in the light-cone[8],

$$\phi_\pi(x, Q^2) = \int_0^{Q^2} d^2k_\perp \; \psi_{q\bar{q}/\pi}(x, k_\perp) \quad . \tag{11}$$

Usually, one assumes that higher Fock state contributions are suppressed by v with respect to that from the valence Fock state $\mid (c\bar{c})_1 >$. It was shown[6] this suppression for inclusive decays of χ_{cJ} does not hold and the contrbution from the color-octet $\mid (c\bar{c})_8(^3S_1)g >$ (i.e. the one arising from the higher Fock state $\mid c\bar{c}g >$ with $c\bar{c}$ in a lolor-octet state) in Eq.(5) is not suppressed by v at all. Fothermore, Ref.[9] argue that for exclusive χ_{cJ} decays the color-octet contribution is not suppressed by powers of either v or $\frac{1}{M_c}$ too.

After taking into account for the higher Fock state contribution we can write down the decay width of the process $\chi_{cJ} \to \pi\pi$,

$$\Gamma(\chi_{cJ} \to \pi\pi) = \Gamma^{(1)}(\chi_{cJ} \to \pi\pi) + \Gamma^{(8)}(\chi_{cJ} \to \pi\pi) \quad , \tag{12}$$

where $\Gamma^{(8)}(\chi_{cJ} \to \pi\pi)$ is determined by the color-octet contribution. It will be estimated by the wave function of the color-octet $\mid (c\bar{c})_8 g >$ and there are more uncertainties to calculate the color-octet contribution.

2. Phenomenological analysis to the decay widths

It is shown that the color octet of χ_{cJ} will make contributions to the decay width of them. However we haven't a good framework to calculate their contributions precisely yet at present. In order to estimate them phenomenologically we rewrite the decay widths of χ_{cJ} as

$$\Gamma(\chi_{cJ}) = \Gamma^{(1)}(\chi_{cJ})[1 + \Delta_J] \tag{13}$$

$$\Gamma(\chi_{cJ} \to \pi\pi) = \Gamma^{(1)}(\chi_{cJ} \to \pi\pi)[1 + a_J^\pi] \tag{14}$$

$$\Gamma(\chi_{cJ} \to KK) = \Gamma^{(1)}(\chi_{cJ} \to KK)[1 + a_J^K] \tag{15}$$

where $\Gamma^{(1)}(\chi_{cJ}), \Gamma^{(1)}(\chi_{cJ} \to \pi\pi)$ and $\Gamma^{(1)}(\chi_{cJ} \to KK)$ are determined by the color-singlet state of χ_{cJ}. All of contributions from the color-octet state of χ_{cJ} are included in the parameters Δ_J, a_J^π and a_J^K. $\Gamma^{(1)}(\chi_{c0}$ and $\Gamma^{(1)}(\chi_{c2}$ are determined by Eqs.(1) and (2). Then we have the raio R

$$\begin{aligned} R \equiv \frac{\Gamma_{tot}(\chi_{CO})}{\Gamma_{tot}(\chi_{c2})} &= \frac{\Gamma_{tot}^{(1)}(\chi_{CO})(1+\Delta_0)}{\Gamma_{tot}^{(1)}(\chi_{c2})(1+\Delta_2)} \\ &= \frac{15}{4} \; \frac{(1+\Delta_0)}{(1+\Delta_2)} \end{aligned} \tag{16}$$

and the branching ratios

$$Br(\chi_{c0} \to \pi^+\pi^-) = \frac{8\pi^2}{81} \; \frac{\alpha_s^2(M_c^2)}{M_c^4} \mid I_0^\pi \mid^2 \frac{1+a_0^\pi}{1+\Delta_0} \tag{17}$$

$$Br(\chi_{c2} \to \pi^+\pi^-) = \frac{4\pi^2}{27} \; \frac{\alpha_s^2(M_c^2)}{M_c^4} \mid I_2^\pi \mid^2 \frac{1+a_2^\pi}{1+\Delta_2} \tag{18}$$

$$Br(\chi_{c0} \to K^+K^-) = \frac{8\pi^2}{81} \; \frac{\alpha_s^2(M_c^2)}{M_c^4} \mid I_0^K \mid^2 \frac{1+a_0^K}{1+\Delta_0} \tag{19}$$

$$Br(\chi_{c2} \to K^+K^-) = \frac{4\pi^2}{27} \; \frac{\alpha_s^2(M_c^2)}{M_c^4} \mid I_2^K \mid^2 \frac{1+a_2^K}{1+\Delta_2} \quad . \tag{20}$$

Thus we eliminate the uncertainty of $R_p^{'}(0)$ in Eqs.(16-20). In fact, the parameters Δ_J, a_J^π and a_J^K may include all of contributions from the higher Fock state and the higher order terms. From the experiments

$$\begin{aligned} \Gamma_{tot}(\chi_{c0}) &= 14.3 \pm 2.0 \pm 3.0 \; MeV \\ \Gamma_{tot}(\chi_{c2}) &= 2.00 \pm 0.18 \; MeV \end{aligned}$$

follows

$$\frac{1+\Delta_0}{1+\Delta_2} \sim 1 - 3 \tag{21}$$

which leads an inequality

$$\Delta_0 > \Delta_2 \tag{22}$$

Recent BES measurement gets the ratios of the branching fractions,

$$\frac{Br(\chi_{c0} \to \pi^+\pi^-)}{Br(\chi_{c0} \to K^+K^-)} = 0.82 \pm 0.15$$

and

$$\frac{Br(\chi_{c2} \to \pi^+\pi^-)}{Br(\chi_{c2} \to K^+K^-)} = 1.88 \pm 0.51$$

Comparing the experimental data with Eqs.(17-20), we can obtain some constrants on the parameters a_J^π and a_J^K,

$$\frac{\mid I_0^\pi \mid^2 (1 + a_0^\pi)}{\mid I_0^K \mid^2 (1 + a_0^K)} = 0.82 \pm 0.15 \tag{23}$$

$$\frac{\mid I_2^\pi \mid^2 (1 + a_2^\pi)}{\mid I_2^K \mid^2 (1 + a_2^K)} = 1.88 \pm 0.51 \tag{24}$$

If we ignore the SU(3) symmetry breaking of the π and K wave functions, Eqs.(23) and (24) tell us one constraint

$$a_0^\pi \simeq a_0^K \tag{25}$$

for the χ_{c0} state and another constraint

$$a_2^\pi > a_2^K \tag{26}$$

for the χ_{c2} state. Therefore the present experimental data have put some constraint on the color-octet contributions. In order to fit the BES data it requires that (1) the contributions to the χ_{c0} inclusive decay are larger than the contributions to χ_2 from the higher Fock states i.e. $\Delta_0 > \Delta_2$; (2) For the χ_{c0} state, the corrections to the process $\chi_{c0} \to \pi\pi$ from the higher Fock states is approximately equal to the correcitons to the process $\chi_{c0} \to KK$; (3) For the χ_{c2} states, the corrections to the process $\chi_{c2} \to \pi\pi$ is larger than the corrections to the process $\chi_2 \to KK$ from the higher Fock states.

3. Numerical estimate to the valence and hgiher Fock state contribution

Now let us calculate the valence Fock state contribution, such as $\Gamma^{(1)}(\chi_{cJ})$, $\Gamma^{(1)}(\chi_{cJ} \to \pi\pi)$ and $\Gamma^{(1)}(\chi_{cJ} \to KK)$ in the leading order. The results, according to the Eqs.(7-9), are all dependent on the distribution amplitude $\phi(x, Q^2)$ of the pion and kaon which are independent of the processes apart from the energy scale Q^2. the $\phi_M(x, Q^2)$ of the meson is defined by Eq.(11) and $\psi_{q\bar{q}/M}(x, k_\perp)$ is the valance wavefunction of the meson in the light-cone framework. The general solution of the QCD evolution equation[8]

$$\phi_M(x, Q^2) = x(1-x) \sum_{n=0}^{\infty} a_n(Q_0^2) C_n^{3/2}(2x-1) (\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)})^{\gamma_n} \tag{27}$$

and the coefficients $a_n(Q_0^2)$ are dependent upon the initial distribution amplitude $\phi_M(x, Q_0^2)$ through its definition

$$a_n(Q_0^2) = \frac{2(2n+3)}{(2+n)(1+n)} \int_{-1}^{1} d(2x-1) C_n^{3/2}(2x-1) \phi_\pi(x, Q_0^2) \tag{28}$$

with $a_0 = \sqrt{3} f_M$. f_M is the decay constant of the meson. As $Q^2 \to \infty, \phi_M(x, Q^2)$ goes to the asymptotic form $\phi_M^{as}(x)$

$$\phi_M^{as}(x) = \sqrt{3}\ f_M x(1-x) \quad . \tag{29}$$

If employing the asymptotic form, one finds the asymptotic values of the branching ratios of the χ_{cJ} are much smaller than the experimental values. In order to fit the data one possible solution is to take a much wider distribution amplitude[10]. For examle, from the Chernyak-Zhitnisky (CZ) form[11]

$$\phi_M(x) = \frac{f_M}{\sqrt{3}} 15(2x-1)^2 x(1-x) \tag{30}$$

the valence Fock state contribution can be consistant with experimental values very well without any color-octet contribution. However the recent studies on the processes involving the pion disfavor the CZ distribution amplitude[12−15]. It follows that the pion distribution amplitude is close to the asymptotic form. Following Refs.[12,16], we take the model wavefunction

$$\psi_\pi(x, k_\perp) = A_\pi \quad exp[-\frac{k_\perp^2 + m^2}{8\beta^2 x(1-x)}] \tag{31}$$

and

$$\psi_K(x,k_\perp) = A_K \;\; exp[-(\frac{k_\perp^2+m^2}{8\beta^2 x} + \frac{k_\perp^2+m_s^2}{8\beta^2(1-x)})] \tag{32}$$

with $A_\pi = 32GeV^{-1}, A_K = 52.6GeV^{-2}, \beta = 385MeV, m = 298MeV$ and $m_s = 550MeV$. It leads the distribution amplitudes

$$\phi_\pi(x) = \frac{2\beta^2 A_\pi}{(2\pi)^2} x(1-x)\; exp(-\frac{m^2}{8\beta^2 x(1-x)}) \tag{33}$$

and

$$\phi_K(x) = \frac{2\beta^2 A_K}{(2\pi)^2} x(1-x)\; exp(-\frac{1}{8\beta^2}(\frac{m^2}{x} + \frac{m_s^2}{1-x})) \tag{34}$$

Substituting Eqs.(33-34) into (7-9, 17-20) we can get the following numerical results

$$Br(\chi_{c0} \to \pi^+\pi^-) \;=\; B_0(0.1109)^2 \frac{1+a_0^\pi}{1+\Delta_0} \tag{35}$$

$$Br(\chi_{c2} \to \pi^+\pi^-) \;=\; B_2(0.4538 \times 10^{-1})^2 \frac{1+a_2^\pi}{1+\Delta_2} \tag{36}$$

$$Br(\chi_{c0} \to K^+K^-) \;=\; B_0(0.1256)^2 \frac{1+a_0^K}{1+\Delta_0} \tag{37}$$

$$Br(\chi_{c2} \to K^+K^-) \;=\; B_2(0.5329 \times 10^{-1})^2 \frac{1+a_2^K}{1+\Delta_2} \tag{38}$$

where

$$B_0 = \frac{8\pi^2}{81} \;\; \frac{\alpha_s^2(M_c^2)}{M_c^4(GeV)^{-4}}, \quad B_2 = \frac{4\pi^2}{27} \;\; \frac{\alpha_s^2(M_c^2)}{M_c^4(GeV)^{-4}} \tag{39}$$

From Eqs.(35-38) follows

$$\frac{Br(\chi_{c0} \to \pi^+\pi^-)}{Br(\chi_{c0} \to K^+K^-)} = 0.78 \frac{1+a_0^\pi}{1+a_0^K} \tag{40}$$

and

$$\frac{Br(\chi_2 \to \pi^+\pi^-)}{Br(\chi_2 \to K^+K^-)} = 0.73 \frac{1+a_2^\pi}{1+a_2^K} \tag{41}$$

These ratios are independent of the parameters $\alpha_s(M_c)$ and M_c. Comparing Eqs.(40-41) with BES experimental values one obtains

$$a_0^\pi \simeq a_0^K \tag{42}$$

$$a_2^\pi \cong 2.58 \ \ a_2^K + 1.58 \tag{43}$$

which means that the higher Fock state contribution depends on the total spin J of the p-wave state χ_{cJ}. Eqs.(42-43) show that a_2^π is rather larger than a_2^K.

The precise prediction will be determined by the parameters $\alpha_s(M_c), R_p'(0), M_c, \Delta_J, a_J^\pi$ and a_J^K. In this paper we only apply the phenomenological analysis to the branching ratios to eliminate the parameters $\alpha_s(M_c), R_p'(0)$ and M_c for getting some constraints on the parameters Δ_J, a_J^π and a_J^K .

4. Conclusion

We have presented a phenomenological analysis of higher Fock state contributions to the χ_{cJ} decays by using the recent BES experimental data. In this paper we include the higher Fock state contributions to the decays of χ_{cJ} beyond the valence Fock state contribution. We parameterize the higher Fock state and higher order contributions as Δ_J for inclfusive processes and a_J^π and a_J^K for exclusive processes of the χ_{cJ}. The recent experimental data requires that (i) For the inclusive process of the χ_{cJ}, the higher Fock state $\mid (c\bar{c})_8 g >$ makes an important contribution with respect to that from the valance Fock state $\mid c\bar{c} >$ and they are different for the χ_{c0} and χ_{c2} state, i.e. $\Delta_0 > \Delta_2$. (ii) The similar results are obtained for the exclusive processes and we find $a_0^\pi \simeq a_0^K$ for the χ_{c0} state and $a_2^\pi > a_2^K$ for the χ_{c2} state.

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